Quantitative Literacy:
Thinking Between the Lines

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Chapter 4:
Personal Finance

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Lesson Plan

- Saving money: The power of compounding
- Borrowing: How much car can you afford?
- Savings for the long term: Build that nest egg
- Credit cards: Paying off consumer debt
- Inflation, taxes, and stocks: Managing your money

Learning Objectives:

- Use the simple interest and compound interest formulas
- Compute Annual Percentage Yield (APY)
- Understand and calculate the Present value and the Future value
- Compute the exact doubling time
- Estimate the doubling time using the Rule of 72

4.1 Saving money: The power of compounding

- **Principal:** The initial balance of an account.
- **Simple interest:** calculated by the interest rate to the principal only, not to interest earned.

Simple Interest Formula

Simple interest earned = Principal × Yearly interest rate × Time

- **Example:** We invest $2000 in an account that pays simple interest of 4% each year. Find the interest earned after five years.

- **Solution:** The interest rate of 4% written as a decimal is 0.04.

Simple interest earned

= Principal × Yearly interest rate × Time in years

= $2000 × 0.04 /year × 5 years

= $400

- **APR (Annual Percentage Rate)** – multiply the period interest rate by the number of periods in a year.

APR Formula

$$\text{Period interest rate} = \frac{\text{APR}}{\text{Number of periods in a year}}$$

- **Compound interest** – interest paid on **both** the principal and on the interest that the account has earned.

Compound Interest Formula

$$\text{Balance after } t \text{ periods} = \text{Principal} \times (1 + r)^t$$

- **Example:** Suppose we invest $10,000 in a five-year certificate of deposit (CD) that pays an APR of 6%. What is the value of the mature CD if interest is:

1. compounded **annually**?
2. compounded **quarterly**?
3. compounded **monthly**?
4. compounded **daily**?

Solution:

1. Compounded **annually:** the rate is the same as the APR:

$$r = 6\% = 0.06 \text{ and } t = 5\ years.$$

$$\text{Balance after 5 years} = \text{Principal} \times (1 + r)^t$$
$$= \$10000 \times (1 + 0.06)^5$$
$$= \$10{,}000 \times 1.06^5 = \$13{,}382.26$$

Solution:

2. Compounded **quarterly:**

$$r = \text{Quarterly rate} = \frac{APR}{4} = \frac{0.06}{4} = 0.015.$$

5 years is 5 × 4 quarters, so $t = 20$ in compound interest formula:

$$\text{Balance after 5 years} = \text{Principal} \times (1 + r)^t$$
$$= \$10{,}000 \times (1 + 0.015)^{20}$$
$$= \$13{,}468.55$$

Solution:

3. Compounded **monthly:**

$$r = \text{Monthly rate} = \frac{APR}{12} = \frac{0.06}{12} = 0.005$$

5 years is 5 × 12 months, so $t = 60$:

$$\begin{aligned}\text{Balance after 5 years} &= \text{Principal} \times (1 + r)^t \\ &= \$10{,}000 \times (1 + 0.005)^{60} \\ &= \$13{,}488.50\end{aligned}$$

Solution:

4. Compounded **daily:**

$$r = \text{daily rate} = \frac{APR}{365} = \frac{0.06}{365} = 0.00016.$$

5 years is 5 × 365 = 1825 days.

$$\begin{aligned}\text{Balance after 5 years} &= \text{Principal} \times (1 + r)^t \\ &= \$10{,}000 \times 1.00016^{1825} \\ &= \$13{,}498.26\end{aligned}$$

Solution:

We summarize the results.

Compounding period	Balance at maturity
Yearly	$13,382.26
Quarterly	$13,468.55
Monthly	$13,488.50
Daily	$13,498.26

4.1 Saving money: The power of compounding

- The annual percentage yield (**APY**) – the actual percentage return earned in a year.

$$\text{APY} = \left(1 + \frac{APR}{n}\right)^{n} - 1$$

Where *n* is the number of compounding periods per year.

- **Example:** You have an account that pays APR of 10%. If interest is compounded monthly, find the APY.

- **Solution:** APR = 10% = 0.10, n = 12 , so we use the APY formula:

$$\text{APY} = \left(1 + \frac{APR}{n}\right)^{n} - 1 = \left(1 + \frac{0.10}{12}\right)^{12} - 1 = 0.1047$$

Round the answer as a percentage to two decimal places; the APY is about 10.47%.

APY Balance Formula

Balance after t years = Principal $\times (1 + APY)^t$

- **Example:** Suppose we earn 3.6% APY on a 10-year $100,000 CD. Find the balance at maturity.
- **Solution:** APY = 3.6% = 0.036, t = 10, and so we use APY balance formula:

$$\text{Balance after 10 years} = \$100{,}000 \times (1 + 0.036)^{10}$$
$$= \$142{,}428.71$$

- **Present value:** the amount we initially invest.

Present value = Principal

- **Future value:** the value of that investment at some specific time in the future.

Future value = balance after t periods

Compound Interest Formula

$$\text{Balance after } t \text{ periods} = \text{Principal} \times (1 + r)^t$$

$$\text{Future value} = \text{Present value} \times (1 + r)^t$$

- **Example:** Find the future value of an account after three years if the present value is $900, the APR is 8%, and interest is compounded quarterly.

- **Solution:** $r = \frac{APR}{4} = \frac{.08}{4} = 0.02, \quad t = 3 \times 4 = 12$:

$$\begin{aligned}\text{Future value} &= \text{Present value} \times (1 + r)^t \\ &= \$900 \times (1 + 0.02)^{12} \\ &= \$1141.42\end{aligned}$$

- **Exact doubling time** for investments

$$\text{Number of periods to double} = \frac{\log 2}{\log(1+r)}$$

where r is the period interest rate as a decimal.

- **Approximate doubling time** using *the rule of 72*

$$\text{Estimate for doubling time} = \frac{72}{\text{APR}}$$

where APR is expressed as a **percentage.**

- **Example:** Suppose an account has an APR of 8% compounded quarterly. Estimate the doubling time using the rule of 72. Calculate the exact doubling time.
- **Solution:** The rule of 72 gives the estimate doubling time 9 years.

$$\text{Estimate for doubling time} = \frac{72}{\text{APR}} = \frac{72}{8} = 9 \text{ years}$$

To find the exact doubling time, $r = \frac{}{4} = 0.02$, since the period is quarterly.

$$\text{Number of periods to double} = \frac{\log 2}{\log(1 + 0.02)} = 35.0$$

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months.

Chapter 4 Personal Finance: **Chapter Summary**

- **Savings:** simple interest or compound interest
  - Formulas: simple interest earned
    period interest rate
    balance after *t* periods
    APY
    Present value or Future value
    Number of periods to double
- **Borrowing:** an installment loan
  - Formulas: Monthly payment
    Amount borrowed
  - Fixed-rate mortgage vs. ARM

Chapter 4 Personal Finance: **Chapter Summary**

- Saving for the long term: Build the nest egg (Annuity)
  - Formulas: Balance after *t* deposits
    Needed deposit
    Monthly annuity yield
    Nest egg needed
- Credit cards
  - Formulas: Amount subject to finance charges
    Balance after *t* minimum payments
- Inflation, taxes, and stocks
  - Understand CPI, taxes, DJIA